Independent events:

Example: 2 people each roll a die, what is prob of 2 sixes?

Combine into a single event to see rule

\[ \frac{1}{36} \text{ possible events} \]

General rule for independent outcomes

\[ P(\text{ANB}) = P_A \times P_B \]

Be careful depending on question. What about prob of \( \{5, 6\} \)?

\[ P(\text{person1 - 6 } \text{ and } \text{person2 - 5} = \frac{1}{36} \text{ so prob of } \]

but also opposite case = \( \frac{1}{36} \), \( 11 \) scoring \( 11 \)

\[ \text{or } \]

\[ \frac{2}{36} = \frac{1}{18} \]

(this is an example of the "or" rule for the joint outcomes, add area)

What about prob that one rolls 5 or the other rolls 5 (either, not both)
What about \( A = 2 \) or \( B = 6 \)

but not \( A = B = 2 \)

\[
P_{A=2 \cap B=6} = P_{A=2} + P_{B=6} - P_{A=2 \cap B=2} - P_{A=6 \cap B=6}
\]

Not "\( \sim \)" prob event \( X \) doesn't happen = \( 1 - P_X \)
For a sequence of observations \( \xi_{0;3} = \{x_1, x_2, x_3 \ldots x_n\} \)

\[ D_0 = x_1 \land o_2 = x_2 \land \ldots \land o_N = x_N = \prod_{i=1}^{N} P_{Xi} \]

[Worksheet, 5 min]

How does order matter?

→ How many ways to rearrange \( N \) objects → Turns out:

\[ N! = N(N-1)(N-2) \ldots 1 \]

This is because, imagine \( N \) slots

First item has \( N \) choices, Second item \( N-1 \), and so forth, until all
Returning to coin flips, a sequence would be like H, T, T, H, T, ...

prob would be \( P_{01} = P_H P_T P_T P_H P_T \)...

Every seq is unique\( = P_{H}^{NH} P_{T}^{NT} \)

\[ \Rightarrow = P_H^{NH} (1-P_H)^{N-\text{NH}} \]

mutual exclusive

What if we just want to know how many seqs of length \( N \) have \( NH \) heads then these could come in any order, and the prob of \( NH/N \) is much higher

What is \# ways to order \( NH \& NT \) items?

ordering coins in \( N \) slots

w.l.o.g. Put in \( NH \) coins, as before

\( N(N-1) \cdots (N-\text{NH}) \cdots \) but only down to \( N-N_H \)

\( G = \frac{N!}{NH!} \) In every case there are \( NT \) slots

Filled w/ T's, which can go in \( NT! \) order

Each of these sequences is identical if indistinguishible so

\( \binom{N}{NT} = \frac{N!}{NH! \cdot NT!} = \frac{N!}{NH!(N-N_H)!} = \frac{N!}{(N_T!)(N-N_T)!} \)
These values, written \( \binom{N}{M} \) or \( N \choose M \), are called “binomial” coefficients b/c they are the terms in expansion

\[
(a + b)^N = a^N + \binom{N}{1}a^{N-1}b + \ldots = \sum_{i=0}^{N} \binom{N}{i}a^{N-i}b^i
\]

\[
\text{Prob } (N_A; N) = \binom{N}{N_A} P_A^{N_A} (1-P_A)^{N-N_A}
\]

\( \text{Binomial distribution} \)

\[
\text{Normalized: } \sum_{N_A=1}^{N} \binom{N}{N_A} P_A^{N_A} P_B^{N-N_A} = (P_A + P_B)^N = 1^N = 1 \checkmark
\]

\[\text{Familiar terms: } 1 \ 2 \ 1 \in \text{ Pascal's triangle} \]

\[1 \ 3 \ 3 \ 1 \text{ triangle} \]

\( \text{Key: Meaning is, probability of exactly } m \text{ successes in } N \text{ trials } (\text{Binom}(N,m)) \)

\[
p(m) \propto p^{m/2}
\]

\[
p(m) \propto p^{N/2}
\]
Mean and variance

Mean is simple average

Know, if \( \{X_1, X_2, X_3, \ldots \} \), average is

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{< this is a sample mean, it is computed from data}
\]

If we have a distribution of "X's"

\[
\mu = \frac{1}{n} \sum_{i=1}^{n} X_i \quad P(X_i) \quad \text{also written } \langle X \rangle
\]

Another important quantity is variance, \( \sigma^2 \)

\[
\sigma^2 = \langle (X - \langle X \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2 = \sum_{i=1}^{n} (X_i - \mu)^2 P(X_i)
\]

For data \( \sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{x})^2 \) explain

assumed \( X_i \) is "sampled" from dist \( \mathcal{D} \)

Binomial dist

\[
\mu = Np \quad \sigma^2 = Np(1-p)
\]

\( \Rightarrow \frac{\sigma}{N} \sim \frac{1}{\sqrt{N}} \) so dist gets more rel narrow
Poisson Distribution

Key: prob of a number of random events happens in fixed interval (usually time)
Like number of decay events of radioactive nuclei per hour or number of proteins in some area in a membrane.

Comes from Binom $N \rightarrow \infty$ trials of rare events.

$p(n, \mu) = \frac{\mu^n e^{-\mu}}{n!}$ where $\mu$ is avg number expected.

Eg: $\sigma_{protein} = \frac{1}{\mu m^2}$ look at 100 nm² area

$\mu = \sigma A = \frac{100 mm^2}{1x10^6 nm^2} = 1x10^{-4}$

most likely 0! 

$\sigma^2 = \mu$ 

$\sigma/\mu \sim \sqrt{\mu}$ also